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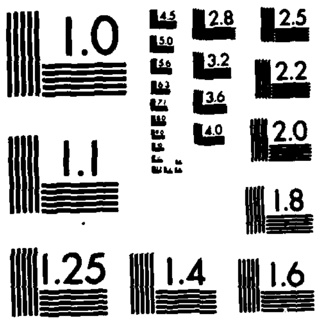
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FOREIGN TECHNOLOGY DIVISION

DYNAMICS OF ANGULAR MOVEMENTS OF A SOLID SUPPORTING A ROTATING ROTOR WITH CONSIDERATION OF ENERGY DISSIPATION

by

V. A. Grobov, I. I. Kantemir

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DYNAMICS OF ANGULAR MOVEMENTS OF A SOLID SUPPORTING A ROTATING ROTOR WITH CONSIDERATION OF ENERGY DISSIPATION

V. A. Grobov, I. I. Kantemir

This article analyzes the motion of a system consisting of an axisymmetric solid B with mass M, rotor B' with mass M', and two passive dampers b and b', each of which consists of a spring and mass placed into a tube filled with a viscous fluid. The schematic of the system is shown in the figure. Body B is used as the principal body, i.e., a body relative to which the movement of all the components of the system is being analyzed. X_1, X_2, X_3 is the coordinate system coupled with body B, the origin of which is at the center of masses of body B and one of its axes - X_3 - is oriented along the symmetry axis of the body. Consequently, axes X_1, X_2, and X_3 are the principal central axes of inertia of body B. The origin of the $X_1'X_2'X_3'$ system coupled with rotor B' is at the center of masses $B^{*'}$; axis X_3' coincides with axis X_3. The distance between $B^{*'}$ and B^* equals l, while ψ is the angle between axis X_1' and the line, which is parallel to axis X_1 and passes through point $B^{*'}$.

The damping mechanism consists of mass m, which moves in the tube filled with viscous fluid. We designate the coefficients of the spring's rigidity and viscous damping of the fluid by k and c. The tube is attached to body B in parallel to axis X_3 in such a way that the spring is not stretched when particle m is at axis X_1 at distance a from point B^*. The displacement of the particle relative to axis X_1 is determined

by value z. Three additional particles, the mass of each of which is equal to the total mass of damper m_b, are coupled rigidly with body B in such a way that as to have the system (body, damper, and three particles) symmetrical relative to axis X_3 when the spring is not stretched. A similar damper is attached also to rotor B' [4].

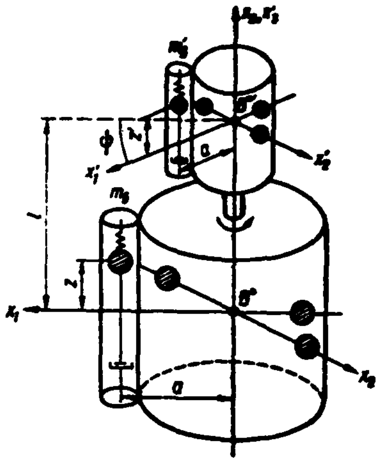

Fig. 1.

Let the $\omega_1, \omega_2, \omega_3$ be the angular velocities of rotation of body B around axes X_1, X_2, and X_3 in an inertial frame of reference. We assume that the angular rotation speed of the rotor around axis X_3 equals $\omega_3 + \sigma$, where σ=const.

To formulate the equations of motion, first we will work out the expressions for the kinetic and potential energy of the system.

The kinetic energy of the system equals

$$
\begin{aligned}
T = \frac{1}{2}\{(A - M_T\zeta^2 - 2M_T\zeta\nu l + m_b z^2 + m_b' z'^2 + 2z' l m_b')(\omega_1^2 + \omega_2^2) + \\
+ C\omega_3^2 + 2J_3'\sigma\omega_3 + J_3'\sigma^2 + m_b(\dot{z}^2 - 2a\omega_1\omega_3 z - 2a\dot{\omega}_2 z) + \\
+ m_b'[\dot{z}'^2 - 2a'\dot{z}'(\omega_3 + \sigma)(\omega_1 \cos\psi + \omega_2 \sin\psi) + 2a'z'\dot{\omega}_1 \sin\psi - \\
- 2a'z'\dot{\omega}_2 \cos\psi] - M_T\dot{\zeta}^2\},
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
A = I_1 + I_1' + 2m_b a^2 + 2m_b' a'^2 + (M' + 4m_b')(1 - \nu) l^2, \\
C = I_3 + I_3' + 4m_b a^2 + 4m_b' a'^2, \\
J_3' = I_3' + 4m_b' a'^2, \\
\nu = (M' + 4m_b')/(M + M' + 4m_b + 4m_b'), \\
M_T = M + 4m_b + M' + 4m_b', \\
p = \frac{m_b}{M_T}, \quad p' = \frac{m_b'}{M_T}, \quad \zeta = pz + p'z',
\end{aligned}
\tag{2}
$$

I_1, I_3 and I_1', I_3' are the principal enertia moments of body B and rotor B', respectively.

The potential energy of the system has the form

$$\Pi = \frac{1}{2}(kz^2 + k'z'^2). \tag{3}$$

Taking into account expressions (1) and (3), we write the equations of motion in the Lagrange form:

$$\begin{gathered}
A\dot{\omega}_1 - (A - C)\omega_2\omega_3 + J_3'\sigma\omega_2 + 2M_T\zeta\dot{\zeta}\omega_1 + M_T\zeta^2(\dot{\omega}_1 - \omega_2\omega_3) + \\
+ m_b\{-2(\zeta + l_2)[z(\dot{\omega}_1 - \omega_2\omega_3) + \dot{z}\omega_1] + z[-2\dot{\zeta}\omega_1 + 2\dot{z}\omega_1 + \\
+ z(\dot{\omega}_1 - \omega_2\omega_3) - a(\dot{\omega}_3 + \omega_1\omega_2)]\} + m_b'[-2(\zeta - l_1)[z'(\dot{\omega}_1 - \omega_2\omega_3) + \\
+ \dot{z}'\omega_1] + z'[-2\dot{\zeta}\omega_1 + 2\dot{z}'\omega_1 + z'(\dot{\omega}_1 - \omega_2\omega_3) - \\
- a'\cos\psi(\dot{\omega}_3 + \omega_1\omega_2)] + a'\sin\psi\{\ddot{z} + z'[(\omega_3 + \sigma)^2 - \omega_2^2]\}] = 0,
\end{gathered} \tag{4a}$$

$$\begin{gathered}
A\dot{\omega}_2 - (C - A)\omega_1\omega_3 - J_3'\sigma\omega_1 + 2M_T\zeta\dot{\zeta}\omega_2 + M_T\zeta^2(\dot{\omega}_2 + \omega_1\omega_3) + \\
+ m_b\{-2(\zeta + l_2)[z(\dot{\omega}_2 + \omega_1\omega_3) + \dot{z}\omega_2] + z[-2\dot{\zeta}\omega_2 + 2\dot{z}\omega_2 + \\
+ z(\dot{\omega}_2 + \omega_1\omega_3)] - a[\ddot{z} + z(\omega_3^2 - \omega_1^2)]\} + m_b'\left[-2(\zeta - l_1)[z'(\dot{\omega}_2 + \omega_1\omega_3) + \right. \\
+ \dot{z}'\omega_2] + z'[-2\zeta\dot{\omega}_2 + 2\dot{z}'\omega_2 + z'(\dot{\omega}_2 + \omega_1\omega_3)] - \\
\left. - a'\cos\psi\{\ddot{z}' + z'[(\omega_3 + \sigma)^2 - \omega_1^2]\} - a'z'\sin\psi(\dot{\omega}_3 - \omega_1\omega_2)\right] = 0,
\end{gathered} \tag{4б}$$

$$\begin{gathered}
C\dot{\omega}_3 - m_b a[2\dot{z}\omega_1 + z(\dot{\omega}_1 - \omega_2\omega_3)] - m_b'a'\sin\psi[2\dot{z}'\omega_2 + z'(\dot{\omega}_2 + \omega_1\omega_3)] - \\
- m_b'a'\cos\psi[2\dot{z}'\omega_1 + z'(\dot{\omega}_1 - \omega_2\omega_3)] = 0,
\end{gathered} \tag{4в}$$

$$\begin{gathered}
m_b(1 - p)\ddot{z} - m_b'p\ddot{z}' - m_b a(\dot{\omega}_2 - \omega_1\omega_3) - m_b(\omega_1^2 + \omega_2^2)[z(1 - p) - \\
- l_2 - p'z'] + c\dot{z} + kz = 0,
\end{gathered} \tag{5a}$$

$$\begin{gathered}
- m_b p'\ddot{z} + m_b'(1 - p')\ddot{z}' + m_b'a'\{\sin\psi[\dot{\omega}_1 + \omega_2(\omega_3 + 2\sigma)] - \\
- \cos\psi[\dot{\omega}_2 - \omega_1(\omega_3 + 2\sigma)]\} - m_b'(\omega_1^2 + \omega_2^2)[z'(1 - p') + l_1 - pz) + \\
+ c'\dot{z}' + k'z' = 0.
\end{gathered} \tag{5б}$$

Here we use the designations

$$l_1 = \frac{l(M + 4m_b)}{M_T}, \quad l_2 = \frac{l(M' + 4m_b')}{M_T}. \tag{6}$$

We introduce the following quantity:

$$\Lambda = \frac{(C - A)\omega_3 + J_3'\sigma}{A}, \tag{7}$$

which represents an angular velocity of free precession of a system of bodies. Then equations (4a)–(4c) can be written as

$$\begin{cases}
\dot{\omega}_1 + \Lambda\omega_2 = \mu F_1(\omega_1, \omega_2, \omega_3, \dot{\omega}_1, \dot{\omega}_3, z_1, \dot{z}, z', \dot{z}', \ddot{z}'), \\
\dot{\omega}_2 - \Lambda\omega_1 = \mu F_2(\omega_1, \omega_2, \omega_3, \dot{\omega}_2, \dot{\omega}_3, z, \dot{z}, z', \dot{z}', \ddot{z}'), \\
\dot{\omega}_3 = \mu F_3(\omega_1, \omega_2, \omega_3, \dot{\omega}_1, \dot{\omega}_2, z, \dot{z}, z', \dot{z}'),
\end{cases} \tag{8}$$

where μ is a small parameter:

$$\mu = \frac{m_b' a'^2}{A}, \tag{9}$$

and F_1, F_2, F_3 - nonlinear functions in the form

$$\begin{gathered}
F_1 = \frac{m}{m'a'^2}\{2l_2[z(\dot{\omega}_1 - \omega_2\omega_3) + \dot{z}\omega_1] - 2z\dot{z}\omega_1 - z^2(\dot{\omega}_1 - \omega_2\omega_3) + \\
+ az(\dot{\omega}_3 + \omega_1\omega_2)\} - \frac{1}{a'^2}\{2l_1[z'(\omega_1 - \omega_2\omega_3) + \dot{z}'\omega_1)] + 2z'\dot{z}'\omega_1 + \\
+ z'^2(\dot{\omega}_1 - \omega_2\omega_3) - a'z'\cos\psi(\dot{\omega}_3 + \omega_1\omega_2) + a'\sin\psi[\ddot{z}' + z'((\omega_3 + \sigma)^2 - \omega_2^2)]\}, \\
F_2 = \frac{m}{m'a'^2}\{2l_2[z(\dot{\omega}_2 + \omega_1\omega_3) + \dot{z}\omega_2] - 2z\dot{z}\omega_2 - z^2(\dot{\omega}_2 + \omega_1\omega_3) + \\
+ a[\ddot{z} + z(\omega_3^2 - \omega_1^2)]\} - \frac{1}{a'^2}\{2l_1[z'(\dot{\omega}_2 + \omega_1\omega_3) + \dot{z}'\omega_2] + 2z'\dot{z}'\omega_2 + \\
+ z'^2(\dot{\omega}_2 + \omega_1\omega_3) - a'\cos\psi[\ddot{z}' + z'((\omega_3 + \sigma)^2 - \omega_1^2)] - a'z'\sin\psi(\dot{\omega}_3 - \omega_1\omega_2)\}, \\
F_3 = \frac{A}{Ca'}\left[\frac{am}{a'm'}(2\dot{z}\omega_1 + z\dot{\omega}_1) + (2\dot{z}'\omega_2 + z'\dot{\omega}_2)\sin\psi + (2\dot{z}'\omega_1 + z'\dot{\omega}_1)\cos\psi\right].
\end{gathered} \tag{10}$$

(Here $m_a = m$ и $m_b = m'$.)

After introducing the designations

$$\begin{gathered}
\frac{m'}{M_T} = \mu s_2, \quad \frac{m}{M_T} = \mu s_1, \quad \frac{c}{m} = 2h_1, \quad \frac{c'}{m'} = 2h_2, \\
\frac{k}{m} = \nu_1^2, \quad \frac{k'}{m'} = \nu_2^2, \quad \Phi^2 = \omega_1^2 + \omega_2^2 = \mu r \Phi^2,
\end{gathered} \tag{11}$$

we write equations (5a) and (5b) in the form

$$\begin{cases}
\ddot{z} + 2h_1\dot{z} + \nu_1^2 z = \mu F_4(\Phi^2, \ddot{z}, \ddot{z}', z) + a(\dot{\omega}_2 - \omega_1\omega_3), \\
\ddot{z}' + 2h_2\dot{z}' + \nu_2^2 z' = \mu F_5(\Phi^2, \ddot{z}, \ddot{z}', z') + a'\{\cos\psi[\dot{\omega}_2 - \omega_1(\omega_3 + 2\sigma)] - \\
- \sin\psi[\dot{\omega}_1 + \omega_2(\omega_3 + 2\sigma)]\},
\end{cases} \tag{12}$$

where

$$\begin{gathered}
F_4 = s_1\ddot{z} + s_2\ddot{z}' + \Phi^2 r(z - l_2), \\
F_5 = s_1(\ddot{z} + \ddot{z}') + \Phi^2 r(z' + l_1).
\end{gathered} \tag{13}$$

Thus, as a result of transformations, system (4a)-(5b) is reduced to the system of equations (8) and (12). Resonance cases are possible in this system, when one of the intrinsic frequencies of the system coincides with the frequency of precession. Let us consider some of them, relying on the results of an analysis conducted on the frequency spectrum, which are given in work [4], where it is shown that the following resonance relations are possible in the system:

. Thus, for example, if the following relation is fulfilled between the parameters:

$$\frac{C}{A} = 1 + \left(1 - \frac{J_1}{A}\right)\frac{\sigma}{\omega^2},$$

then there is resonance of the type $\Lambda = \sigma$.

Let us stop to consider one of these cases. Let us assume that $\Lambda = \sigma + \mu\Delta$. Then system (8) can be written as .

$$\begin{cases} \dot{\omega}_1 + \sigma\omega_2 = \mu(F_1 - \Delta\omega_2), \\ \dot{\omega}_2 - \sigma\omega_1 = \mu(F_2 + \Delta\omega_1), \\ \dot{\omega} = \mu F_3. \end{cases} \tag{14}$$

Considering the case of fast rotation of a body around the symmetry axis X_3, i.e., assuming that $\omega_1, \omega_2 \ll \omega_3$, it is possible to consider ω_1^2, ω_2^2 and $\omega_1\omega_2$ to be the value of the order of smallness μ.

In order to solve the system of equations (14) and (12), which describe the motion of this system, we will use the method of perturbations [1-3]. For this, we will first examine an unperturbed system (when μ=0)

$$\begin{cases} \dot{\omega}_1 + \sigma\omega_2 = 0, \\ \dot{\omega}_2 - \sigma\omega_1 = 0, \\ \dot{\omega}_3 = 0 \end{cases} \tag{15}$$

The solution of (15) is found in the form

$$\begin{cases} \omega_1 = \Phi\cos\theta, \\ \omega_2 = \Phi\sin\theta, \\ \omega_0 = \Omega, \end{cases} \tag{16}$$

where $\theta = \sigma t + \delta$.

Using the solutions of (16) as the generating ones, we transform (14) in accordance with the ideas of N. N. Bogolyubov's method of perturbations [1, 2] to the new variables Φ and δ:

$$\begin{cases} \frac{d\Phi}{dt} = \mu(F_1\cos\theta + F_2\sin\theta), \\ \frac{d\delta}{dt} = \frac{\mu}{\Phi}(F_2\cos\theta - F_1\sin\theta) + \mu\Delta, \\ \frac{d\omega_3}{dt} = \mu F_3. \end{cases} \tag{17}$$

When u=0, taking into account (16), system (12) is written as

$$\begin{cases} \ddot{z} + 2h_1\dot{z} + \nu_1^2 z = \Phi a(\sigma - \Omega)\cos\theta, \\ \ddot{z}' + 2h_2\dot{z}' + \nu_2^2 z' = -\Phi a'(\sigma + \Omega)\cos\delta. \end{cases} \tag{18}$$

We introduce the following new variables into (18):

$$z = y_1 e^{-h_1 t}, \quad z' = y_2 e^{-h_2 t}.$$

$$\begin{cases} e^{-h_1 t}\left(\frac{d^2 y_1}{dt^2} + b_1^2 y_1\right) = \Phi a(\sigma - \Omega)\cos\theta, \\ e^{-h_2 t}\left(\frac{d^2 y_2}{dt^2} + b_2^2 y_2\right) = -\Phi a'(\sigma + \Omega)\cos\delta, \end{cases} \quad (19)$$

where

$$b_1^2 = \nu_1^2 - h_1^2, \quad b_2^2 = \nu_2^2 - h_2^2.$$

The solution for (19) we seek in the form

$$\begin{cases} y_1 = A_1^\circ \cos\Omega_1 + H_1 e^{h_1 t}\cos(\theta - \beta_1), \\ y_2 = A_2^\circ \cos\Omega_2 + H_2 e^{h_2 t}\cos\delta, \end{cases} \quad (20)$$

where

$$\Omega_1 = b_1 t + \varphi_1, \quad \Omega_2 = b_2 t + \varphi_2; \quad A_1^\circ = A_1 e^{h_1 t}, \quad A_2^\circ = A_2 e^{h_2 t};$$
$$H_1 = \frac{\Phi a(\sigma - \Omega)}{\sqrt{(\nu_1^2 - \sigma^2)^2 + 4h_1^2\sigma^2}}; \quad \operatorname{tg}\beta_1 = \frac{2h_1\sigma}{\nu_1^2 - \sigma^2};$$
$$H_2 = -\frac{\Phi a'(\sigma + \Omega)}{\nu_2^2}.$$

Let us examine the perturbed system

$$\begin{cases} e^{-h_1 t}\left(\frac{d^2 y_1}{dt^2} + b_1^2 y_1\right) = \mu F_4 + \Phi a(\sigma - \omega_3)\cos\theta, \\ e^{-h_2 t}\left(\frac{d^2 y_2}{dt^2} + b_2^2 y_2\right) = \mu F_5 - \Phi a'(\sigma + \omega_3)\cos\delta. \end{cases} \quad (21)$$

In this case we note that Φ, δ, ω_3, and H_1 and H_2 will be variable in perturbed motion. It is also easy to see that

$$\frac{dH_1}{dt} = \mu Q_1, \quad \frac{dH_2}{dt} = \mu Q_2, \quad \frac{d\delta}{dt} = \mu R_2,$$

where

$$Q_1 = \frac{a(\sigma - \omega_3)}{q_1^2}(F_1\cos\theta + F_2\sin\theta),$$
$$Q_2 = -\frac{a'(\sigma + \omega_3)}{\nu_2^2}(F_1\cos\theta + F_2\sin\theta),$$
$$R = \frac{F_2\cos\theta - F_1\sin\theta}{\Phi} + \Delta.$$

The solution for (21) we seek in form (20), where $A_1^\circ, A_2^\circ, \varphi_1$ and φ_2 satisfy the following system:

$$\begin{cases} \frac{dA_i^\circ}{dt} = \mu\left(\psi_i\cos\Omega_i + \frac{\chi_i}{b_i}\sin\Omega_i\right)e^{h_i t}, \\ \frac{d\varphi_i}{dt} = \mu\frac{1}{A_i^\circ}\left(\frac{\chi_i}{b_i}\cos\Omega_i - \psi_i\sin\Omega_i\right)e^{h_i t} \end{cases} \quad (i = 1, 2). \quad (22)$$

Here we use the designations

$$\psi_1 = H_1 R\sin(\theta - \beta_1) - Q_1\cos(\theta - \beta_1),$$
$$\chi_1 = F_4 + [h_1\cos(\theta - \beta_1) - \sigma\sin(\theta - \beta_1)]Q_1 - H_1[h_1\sin(\theta - \beta_1) + \sigma\cos(\theta - \beta_1)]R.$$

$$\psi_2 = H_2 R \sin\delta - Q_2 \cos\delta,$$
$$\chi_2 = F_2 + h_2 Q_2 \cos\delta - H_2 h_2 R \sin\delta. \qquad (23)$$

Let us now deal with the variables z and z', taking into account that

$$A_i^* = A_i e^{h_i t} \text{ и } \frac{dA_i^*}{dt} = \left(\frac{dA_i}{dt} + A_i h_i\right) e^{h_i t}.$$

Then the solution for system (12) is written as

$$\begin{cases} z = A_1 \cos\Omega_1 + H_1 \cos(\theta - \beta_1), \\ z' = A_2 \cos\Omega_2 + H_2 \cos\delta, \end{cases} \qquad (24)$$

where

$$\begin{cases} \frac{dA_i}{dt} = -A_i h_i + \mu\left(\frac{\gamma_i}{b}\sin\Omega_i + \psi_i \cos\Omega_i\right), \\ \frac{d\varphi_i}{dt} = \frac{\mu}{A_i}\left(\frac{\gamma_i}{b_i}\cos\Omega_i - \psi_i \sin\Omega_i\right) \qquad (i = 1,\ 2). \end{cases} \qquad (25)$$

Equations (17) and (25) are reduced to the standard form. We will make use of the averaging principle for their solution [2, 3]. After averaging the right sides of equations (17) and (25), remaining will be the terms containing only the variables from the slow arguments. The equations of the first approximation relative to the variables Φ, δ, ω_3, A_1, φ_1, A_2 and φ_2 have the following form:

$$\frac{d\Phi}{dt} = -\frac{\mu(\omega_3+\sigma)\Phi}{2}\left[\frac{ma^2(\omega_3-\sigma)^2}{m'a'^2q_1^2}\sin\beta_1 + \frac{(\omega_3+\sigma)^2}{2}\sin 2\delta\right],$$
$$\frac{d\delta}{dt} = -\frac{\mu(\omega_3+\sigma)}{2a'^2}\left[\frac{A_1^2 m}{m'} + A_2^2 + \frac{ma^2(\omega_3-\sigma)^2}{m'q_1^2}\cos\beta_1 - \frac{2a'^2(\omega_3+\sigma)^2}{\nu_2^2}\right] + \mu\Delta,$$
$$\omega_3 = \Omega = \text{const}, \qquad (26)$$
$$\frac{dA_1}{dt} = -A_1 h_1 - \mu\frac{A_1 a^2(\omega_3-\sigma)(b_1^2-\omega_3^2)m}{2m'a'^2}\sin\beta_1,$$
$$\frac{d\varphi_1}{dt} = -\mu\left[\frac{(\omega_3-\sigma)(b_1^2-\omega_3^2)ma^2}{2m'a'^2 b_1 q_1^2}(h_1\sin\beta_1 + \sigma\cos\beta_1) + \frac{b_1 s_1}{2}\right] + \frac{\Phi^2}{2b_1},$$
$$\frac{dA_2}{dt} = -A_2 h_2 - \mu\frac{A_2\Phi(\omega_3+\sigma)l_1}{a'\nu_2^2}[h_2\cos\sigma - (\omega_3+\sigma)\sin\delta],$$
$$\frac{d\varphi_2}{dt} = -\mu\left\{\frac{\Phi(\omega_3+\sigma)l_1}{a'b_2\nu_2^2}[h_2(\omega_3+\sigma)\sin\delta - b_2^2\cos\delta] + \frac{b_2 s_1}{2}\right\} + \frac{\Phi^2}{2b_2},$$

where

$$q_1^2 = \sqrt{(\nu_1^2-\sigma^2)^2 + 4h_1^2\sigma^2}.$$

Taking into account that $\Lambda = \frac{(C-A)\omega_3 + J_3'\sigma}{A}$ and $\Lambda = \sigma = \mu\Delta$, we find

$$\omega_3 + \sigma = \frac{C-J_3'}{A-J_3'}\omega_3 - \mu\frac{\Delta A}{A-J_3'},$$
$$\omega_3 - \sigma = \frac{2A-C-J_3'}{A-J_3'}\omega_3 + \mu\frac{\Delta A}{A-J_3'}. \qquad (27)$$

Substituting (27) in the first equation of (26), with an accuracy to within the value of the order of μ^2, we have

$$\frac{d\Phi}{dt}=-\mu\frac{(C-J_3')\,\Phi\omega_3^2}{2(A-J_3')^2}\left[\frac{(2A-C-J_3')^2\,ma^2}{m'a'^2q_1^2}\sin\beta_1+\frac{(C-J_3')^2}{\nu_2^2}\sin 2\delta\right]. \quad (28)$$

After integrating (28), we obtain

$$\Phi=\Phi_0\exp\left\{-\mu\frac{(C-J_3')\,\omega_3^2}{2(A-J_3')^2}\left[\frac{(2A-C-J_3')^2\,ma^2}{m'a'^2q_1^2}\sin\beta_1(t-t_0)+\right.\right.$$
$$\left.\left.+\frac{(C-J_3')^2}{\nu_2^2}\int_{t_0}^{t}\sin 2\delta dt\right]\right\}. \quad (29)$$

We make an estimate in (19):

$$\Phi<C_0e^{-\mu\frac{(C-J_3')\,\omega_3^2}{2(A-J_3')^2}\left[\frac{(2A-C-J_3')^2ma^2\sin\beta_1}{m'a'^2q_1^2}-\frac{(C-J_3')^2}{\nu_2^2}\right]t}. \quad (30)$$

We note that

$$A-J_3'>0,\quad C-J_3'>0,\quad \sin\beta_1=\frac{2h_1\sigma}{\sqrt{(\nu_1^2-\sigma^2)^2+4h_1^2\sigma^2}}>0.$$

It is obvious that in order for $\Phi(t)\rightarrow 0$, it is sufficient that the following condition is satisfied:

$$\frac{(2A-C-J_3')^2\,ma^2\sin\beta_1}{m'a'^2q_1^2}>\frac{(C-J_3')^2}{\nu_2^2}. \quad (31)$$

Since $(2A-C-J_3')^2-(C-J_3')^2=4\,(A-C)\,(A-J_3')$, the following relation is satisfied when $A>C$:

$$(2A-C-J_3')^2>(C-J_3')^2.$$

Moreover, if

$$2ma^2h_1\sigma\nu_2^2>m'a'^2\,[(\nu_1^2-\sigma^2)^2+4h_1^2\sigma^2],$$

then condition (31) will be satisfied.

Thus, condition (31) is a sufficient condition for the amplitude of the transverse angular motions to attenuate, i.e., to have "true" rotational motion around the X_3 axis. Weaker sufficient conditions are obtained in the form

$$\begin{cases} A>C, \\ 2ma^2h_1\sigma\nu_2^2>m'a'^2\,[(\nu_1^2-\sigma^2)^2+4h_1^2\sigma^2]. \end{cases} \quad (32)$$

For the case $\Lambda=\sigma$, the angular speed of rotation around the X_3 axis is constant and equals Ω. As can be seen from equations (26), in the case where $h_1^2>\omega_1^2$, the oscillatory motions of the mass at the first damper will be attenuating ones. If

$$h_2 + \frac{(\omega_3 - \sigma)(\nu_1^2 - \omega_3^2) m a^2}{2m' a'^3} \sin \beta_1 < 0,$$

then the oscillatory motions will not be attenuating ones. The vibrational motions of the mass at the second damper will be attenuating ones.

When $\Lambda = \nu_1$, the value of ω_3 is not constant but is a certain function of time. In this case the solution of the problem gets more complicated. Due to the cumbersomeness of the expressions for this case, the equations of the first approximation are not presented.

The averaging method makes it possible to reduce the analysis of angular motions of the body supporting the rotor and damping devices to relatively simple equations (26) for the slowly changing amplitudes and phases, the solution of which can be easily obtained with the aid of electronic modeling devices or digital computers and, in certain cases, by direct integration. The analysis of motion makes it possible to point out its following special features.

The movement of the supporting body relative to the center of masses will be close to the Euler-Poinsot motion, at which the angle of nutation, speed of proper rotation, and the amplitude values of the transverse components of angular velocities of the supporting body change slowly with time. The energy of oscillations of elastically suspended masses, excited by the revolution of the supporting body and rotor with a proper selection of the magnitude of masses and parameters of the suspension devices, can be used for damping nutation oscillations of the system.

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